Theory of Multilayer Coating for proof-of-concept experiments

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Nb cavity processed by the ILC recipe

High gradient
Nb cavity (ILC recipe)

T. Kubo et al., IPAC14, WEPR1022
Breakthrough by the nitrogen doping


High $Q$

Nb cavity (Nitrogen doping)

High gradient

Nb cavity (ILC recipe)

A. Romanenko, LINAC14, TUIOC02
T. Kubo et al., IPAC14, WEPR1022
We want to go beyond Nb!

High $Q$

Nitrogen doped Nb cavity

High gradient

ILC recipe

Multilayer is promising

$T = 2 \, [K]$
The multilayer coating was proposed by A. Gurevich, *Appl. Phys. Lett. 88, 012511 (2006)*
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Substrate Material?

Bulk Nb with high RRR?
Nb by magnetron spattering?
The multilayer coating is proposed by A. Gurevich, Appl. Phys. Lett. 88, 012511 (2006).

**How to fix insulator thickness \( d_I \)?**
- 10nm?
- 100nm?

**Substrate Material?**
- Bulk Nb with high RRR?
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The multilayer coating is proposed by A. Gurevich, Appl. Phys. Lett. 88, 012511 (2006).

**Substrate Material?**
- Bulk Nb with high RRR?
- Nb by magnetron spattering?

**How to fix insulator thickness** $d_I$?
- 10nm?
- 100nm?

**How to fix $S$ layer thickness** $d_S$?
- 10nm?
- 100nm?
§ 1

The optimum parameters
1. The magnetic field distribution (and thus the screening current distribution $J \propto dB/dx$) in the $S$ layer is different from the naïve exponential decay.

2. When $d_S$ and $d_I$ are thin enough and $\lambda_1 > \lambda_2$, the screening current in the $S$ layer is suppressed, and the surface field can exceed the superheating field of the $S$ layer.

3. However, an extremely thin $d_S$ can not protect the SC substrate. Thus the $S$ layer must have some thickness to decay the magnetic field and protect the SC substrate. The optimum thickness of $d_S$ exists.

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2. S. Posen et al., in proceedings of SRF2013, p. 788, WEIOC04 [Sep.2013].
Important!

1. The magnetic field distribution (and thus the screening current distribution $J \propto dB/dx$) in the S layer is different from the naïve exponential decay.

![Graph showing magnetic field distribution](image)

Important!

$B(x)/B_0$

$S$ layer

$\lambda = \lambda_1$

$d_S$

$SC$ substrate

$\lambda = \lambda_2$

$d_I$


[submitted to arXiv on April 2013; published on January 2014]

The derivation processes are explained in detail in *proceedings of IPAC13*, p. 2343, WEPWO014 [May 2013]
Important!

1. The magnetic field distribution (and thus the screening current distribution \( J \propto dB/dx \)) in the S layer is different from the naïve exponential decay.

![Graph showing the magnetic field distribution in S and SC layers](image)

The derivation processes are explained in detail in *proceedings of IPAC13*, p. 2343, WEPWO014 [May 2013]

2. When $d_S$ and $d_I$ are thin enough and $\lambda_1 > \lambda_2$, the screening current in the $S$ layer is suppressed, and the surface field can exceed superheating field of the $S$ layer.

The ratio of the field the $S$ layer $B_{\text{max}}^{\text{(S layer)}}$ can withstand to that of the superheating field of $S$ layer $B_s^{\text{(S layer)}}$ is given by:

$$
\frac{B_{\text{max}}^{\text{(S layer)}}}{B_s^{\text{(S layer)}}} = \frac{\cosh \frac{d_S}{\lambda_1} + \left(\frac{\lambda_2}{\lambda_1} + \frac{d_I}{\lambda_1}\right) \sinh \frac{d_S}{\lambda_1}}{\sinh \frac{d_S}{\lambda_1} + \left(\frac{\lambda_2}{\lambda_1} + \frac{d_I}{\lambda_1}\right) \cosh \frac{d_S}{\lambda_1}}
$$

$\frac{d_I}{\lambda_1} = 0.1, \quad \frac{\lambda_2}{\lambda_1} = 0.25$

are assumed here.
3. However, an extremely thin $d_S$ can not protect the SC substrate. Thus the $S$ layer must have some thickness to decay the magnetic field and protect the SC substrate.

\[ B_0 < B_{\text{max}}^{(\text{S layer})} \]

\[ B_i < B_{\text{max}}^{(\text{substrate})} \]
3. However, an extremely thin \( d_S \) can not protect the SC substrate. Thus the S layer must have some thickness to decay the magnetic field and protect the SC substrate.

**Formula for the maximum screening field of the multilayer**

\[
B_{\text{max}}^{(\text{multilayer})} = \begin{cases} 
B_{\text{max}}^{(\text{S layer})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} < B_{\text{max}}^{(\text{substrate})}) \\
\gamma^{-1} \times B_{\text{max}}^{(\text{substrate})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} \geq B_{\text{max}}^{(\text{substrate})}) 
\end{cases}
\]

where

\[
\begin{align*}
B_{\text{max}}^{(\text{S layer})} &= B_s^{(\text{S layer})} \frac{\cosh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_I}{\lambda_1} \right) \sinh \frac{d_s}{\lambda_1}}{\sinh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_I}{\lambda_1} \right) \cosh \frac{d_s}{\lambda_1}} \\
\gamma &= \frac{1}{\cosh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_I}{\lambda_1} \right) \sinh \frac{d_s}{\lambda_1}} \\
B_{\text{max}}^{(\text{substrate})} &= 170 \text{mT} - 240 \text{mT} \quad \text{(if the substrate is Nb)}
\end{align*}
\]

\[B_s^{(\text{S layer})} = 0.84 \, B_c^{(\text{S layer})}\]
3. However, an extremely thin $d_S$ can not protect the SC substrate. Thus the $S$ layer must have some thickness to decay the magnetic field and protect the SC substrate.

**Formula for the optimum thickness of the $S$ layer**

$$d_S = \lambda_1 \ln \left[ \frac{\lambda_1}{\lambda_1 + \lambda_2 + d_I} \frac{B_{s}^{(S\ layer)}}{B_{\text{max}}^{(\text{substrate})}} + \sqrt{\left( \frac{\lambda_1}{\lambda_1 + \lambda_2 + d_I} \frac{B_{s}^{(S\ layer)}}{B_{\text{max}}^{(\text{substrate})}} \right)^2 + \frac{\lambda_1 - \lambda_2 - d_I}{\lambda_1 + \lambda_2 + d_I}} \right]$$

$d_I \leq \mathcal{O}(10)\text{nm}$

T. Kubo (2015),

- The formulae can be derived by using the discussion of A. Gurevich, AIP Advances 5, 017112 (2015) and are described by using the superheating field of the quasi-classical theory and thus valid even at $T \ll T_c$.
- The formulae are generalized version of the Gurevich’s formulae. The formulae includes effects of insulator layer with a finite thickness. **When $d_I \ll \lambda_1$, the formulae are reduced to the Gurevich’s formulae** [A. Gurevich, AIP Advances 5, 017112 (2015)].
Contour plot of $B_{\text{max}}^{(\text{multilayer})}$

Optimum $d_s \sim 70\,\text{nm}$

$d_I$ should be thin. Up to several tens of nm acceptable.

See also A. Gurevich, the 6th international workshop on thin films and new ideas for RF superconductivity, October 2014, Italy and A. Gurevich, AIP Advances 5, 017112 (2015)

$S$ layer: Dirty Nb
$B_c^{(\text{Nb})}=200\,\text{mT}$
$\lambda_1=\lambda^{(\text{dirty Nb})}=180\,\text{nm}$
SC substrate: clean Nb
$B_{\text{max}}^{(\text{Nb})}=240\,\text{mT}$
$\lambda_2=\lambda^{(\text{Nb})}=40\,\text{nm}$

T. Kubo, 2015
§ 2
A further step forward
incorporate non-ideal surfaces
We assume $\xi < \delta < \lambda$. Such small defects almost continuously distribute on the surface.
We assume $\xi < \delta < \lambda$. Such small defects almost continuously distribute on the surface.
The superheating field is suppressed due to the enhanced screening current.

Suppression factor
\[ \tilde{B}_s = \eta B_s \]

\[ \eta = \frac{1}{\alpha} \left( \frac{\Gamma \left( \frac{\alpha}{2} \right) \Gamma \left( \frac{3 - \alpha}{2} \right)}{\sqrt{\pi}} \alpha \sin \frac{\pi (\alpha - 1)}{2} \right)^{\frac{\alpha - 1}{\alpha}} \]

We can evaluate a “suppression factor \( \eta \)” for materials, if we have data of surface topographic studies (see for example “C. Xu et al. Phys. Rev. ST Accel. Beams 14, 123501 (2011)”).

Contour plot of $\eta$

- Steep
- Deep

The formula for the maximum screening field of the multilayer

\[ B_{\text{max}}^{(\text{multilayer})} = \begin{cases} B_{\text{max}}^{(\text{S layer})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} < B_{\text{max}}^{(\text{substrate})}) \\ \gamma^{-1} \times B_{\text{max}}^{(\text{substrate})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} \geq B_{\text{max}}^{(\text{substrate})}) \end{cases} \]

where

\[ B_{\text{max}}^{(\text{S layer})} = B_s^{(\text{S layer})} \frac{\cosh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_l}{\lambda_1} \right) \sinh \frac{d_s}{\lambda_1}}{\sinh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_l}{\lambda_1} \right) \cosh \frac{d_s}{\lambda_1}} \times \eta \]

\[ B_{\text{max}}^{(\text{substrate})} = 170 \text{ mT} - 240 \text{ mT} \]

\[ \gamma = \frac{1}{\cosh \frac{d_s}{\lambda_1} + \left( \frac{\lambda_2}{\lambda_1} + \frac{d_l}{\lambda_1} \right) \sinh \frac{d_s}{\lambda_1}} \]

\[ B_s^{(\text{S layer})} = 0.84 B_c^{(\text{S layer})} \]

\[ \eta \text{ can be estimated by} \]

\[ \eta = \frac{1}{a} \left( \Gamma \left( \frac{\alpha}{2} \right) \Gamma \left( \frac{3 - \alpha}{2} \right) a \sin \frac{\pi (\alpha - 1)}{2} \right) \left( \frac{\alpha - 1}{a} \right) \]

T. Kubo (2015),

The formula for the optimum thickness of the $S$ layer

\[ d_S = \lambda_1 \ln \left[ \frac{\lambda_1}{\lambda_1 + \lambda_2 + d_I} B_{B_{\text{max}}}^{(\text{substrate})} \right] + \sqrt{\left( \frac{\lambda_1}{\lambda_1 + \lambda_2 + d_I} B_{B_{\text{max}}}^{(S \text{ layer})} \right)^2 + \frac{\lambda_1 - \lambda_2 - d_I}{\lambda_1 + \lambda_2 + d_I}} \]

where

\[
\begin{align*}
B_{s}^{(S \text{ layer})} &= 0.84 B_{c}^{(S \text{ layer})} \\
B_{\text{max}}^{(\text{substrate})} &= 170 \text{mT} - 240 \text{mT} \quad \text{(if the substrate is Nb)}
\end{align*}
\]

\[\eta \text{ can be estimated by } \eta = \frac{1}{\alpha} \left( \Gamma \left( \frac{\alpha}{2} \right) \Gamma \left( \frac{3 - \alpha}{2} \right) \alpha \sin \frac{\pi (\alpha - 1)}{2} \frac{\alpha - 1}{\delta} \right)^{\frac{\alpha - 1}{\alpha}}\]
The optimum thickness becomes thin in order to compensate the suppressed superheating field.

- **Optimum** $d_s \sim 170$nm
- **Maximum field** $\sim 470$ mT

**Assumptions**
- **S layer**: $\text{Nb}_3\text{Sn}$ (moderately dirty)
  - $B_{c1}(\text{Nb}_3\text{Sn}) = 540$ mT
  - $\lambda_1 = \lambda(\text{Nb}_3\text{Sn}) = 120$ nm
- **SC substrate**: clean $\text{Nb}$
  - $B_{\text{max}}(\text{Nb}) = B_{c1}(\text{Nb}) = 170$ mT
  - $\lambda_2 = \lambda(\text{Nb}) = 40$ nm

**Contour plots**
- $B_{\text{max}}^{(\text{multilayer})}$
- $\eta = 0.7$
The optimum $S$ layer thickness is a function of

- $\lambda_1$, penetration depth of the $S$ layer
- $\lambda_2$ ($\lambda_1 > \lambda_2$), penetration depth of the substrate
- $d_I$ (< several tens of nm), thickness of the insulator
- $B_{c}^{(S \text{ layer})}$, thermodynamic critical field of the $S$ layer
- $B_{max}^{(\text{substrate})}$ (170-240mT for Nb).
- $\eta$ ($0 < \eta < 1$), superheating field suppression factor

Material parameters and surface topographic studies are necessary in order to obtain the optimum $S$ layer thickness. The defect model and the formula for $\eta$ may be useful to extract $\eta$ from surface topographic studies.
Let us go beyond Nb by using the optimum parameters!

Dirty Nb / \( I / I / \) Nb
- Optimum \( d_s \approx 90\text{nm} \)
- Maximum field \( \approx 250\text{mT} \)

\( \text{Nb}_3\text{Sn} / \( I / I / \) Nb
- Optimum \( d_s \approx 150\text{nm} \)
- Maximum field \( \approx 480\text{mT} \)

\( \text{NbN} / \( I / I / \) Nb
- Optimum \( d_s \approx 130\text{nm} \)
- Maximum field \( \approx 270\text{mT} \)
Appendix for multilayer researchers

The optimum parameters for

**Dirty Nb / I / Nb**
**Nb$_3$Sn / I / Nb**
**NbN / I / Nb**

are given below!
The formula for the maximum screening field

$$B_{\text{max}}^{(\text{multilayer})} = \begin{cases} B_{\text{max}}^{(\text{S layer})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} < B_{\text{max}}^{(\text{substrate})}) \\ \gamma^{-1} \times B_{\text{max}}^{(\text{substrate})} & \text{(if } \gamma B_{\text{max}}^{(\text{S layer})} \geq B_{\text{max}}^{(\text{substrate})}) \end{cases}$$

where

$$B_{\text{max}}^{(\text{S layer})} = B_s^{(\text{S layer})} \frac{\cosh \frac{d_s}{\lambda_1} + \left(\frac{\lambda_2}{\lambda_1} + \frac{d_l}{\lambda_1}\right) \sinh \frac{d_s}{\lambda_1}}{\sinh \frac{d_s}{\lambda_1} + \left(\frac{\lambda_2}{\lambda_1} + \frac{d_l}{\lambda_1}\right) \cosh \frac{d_s}{\lambda_1}} \times \eta$$

$$\eta = \frac{1}{\alpha} \left( \frac{\Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\frac{3-\alpha}{2}\right) \alpha \sin \frac{\pi (\alpha - 1)}{2}}{\sqrt{\pi}} \right) \frac{\frac{\alpha-1}{\alpha}}{\delta}$$

$$B_{\text{max}}^{(\text{substrate})} = 170\,\text{mT} - 240\,\text{mT}$$

$$B_{\text{max}}^{(\text{substrate})} = 0.84 \times 170\,\text{mT}$$
The formula for the optimum thickness of the $S$ layer

$$d_s = \lambda_1 \ln \left[ \frac{\lambda_1 \eta B_s^{(S \text{ layer})}}{\lambda_1 + \lambda_2 + d_I B_{\text{max}}^{(\text{substrate})}} + \sqrt{\left( \frac{\lambda_1 \eta B_s^{(S \text{ layer})}}{\lambda_1 + \lambda_2 + d_I B_{\text{max}}^{(\text{substrate})}} \right)^2 + \frac{\lambda_1 - \lambda_2 - d_I}{\lambda_1 + \lambda_2 + d_I}} \right]_{d_I \leq O(10) \text{nm}}$$

**Necessary parameters**

**$S$ layer:**
- $B_s^{(S \text{ layer})} = 0.84 \times B_c^{(S \text{ layer material})}$
- $\lambda_1 = \lambda^{(S \text{ layer material})}$
- $\eta$ (suppression factor)

**$I$ layer:**
- $d_I \sim 10\text{-}100\text{nm}$

**Substrate:**
- $B_{\text{max}}^{(\text{substrate})} = 170\text{mT} - 240\text{mT}$ (if the substrate is Nb)
- $\lambda_2 = \lambda^{(\text{substrate})} = 40\text{nm}$ (if the substrate is Nb)
\eta = 1
The optimum $d_s$ and the maximum field for **Dirty Nb / I / Nb** system, where Nb substrate is assumed to withstand up to 240, 200, and 170mT.

**Ideal**

- **Dirty Nb**
  - $B_c^{(Nb)}=200$mT
  - $\lambda_1=\lambda^{(dirty Nb)}=180$nm
- SC substrate: clean Nb
  - $B_{max}^{(Nb)}=240$mT
  - $\lambda_2=\lambda^{(Nb)}=40$nm

**Dirty Nb / I / Nb**

- **Dirty Nb**
  - $B_c^{(Nb)}=200$mT
  - $\lambda_1=\lambda^{(dirty Nb)}=180$nm
- SC substrate: clean Nb
  - $B_{max}^{(Nb)}=200$mT
  - $\lambda_2=\lambda^{(Nb)}=40$nm

**Dirty Nb / I / Nb**

- **Dirty Nb**
  - $B_c^{(Nb)}=200$mT
  - $\lambda_1=\lambda^{(dirty Nb)}=180$nm
- SC substrate: clean Nb
  - $B_{max}^{(Nb)}=B_{c1}^{(Nb)}=170$mT
  - $\lambda_2=\lambda^{(Nb)}=40$nm

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Consistent with the Gurevich’s recent result

A. Gurevich, AIP Advances 5, 017112 (2015) and 6th thin film workshop at Italy
The optimum $d_s$ and the maximum field for $\text{Nb}_3\text{Sn} / I / \text{Nb}$ system, where Nb substrate is assumed to withstand up to 240, 200, and 170 mT.

Optimum $d_s \sim 130 \text{nm}$
Maximum field $\sim 500 \text{mT}$

Consistent with the Gurevich’s recent result
A. Gurevich, AIP Advances 5, 017112 (2015) and 6th thin film workshop at Italy

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**assumption**

S layer: $\text{Nb}_3\text{Sn}$ (moderately dirty)
$B_c^{(\text{Nb}_3\text{Sn})}=540 \text{mT}$
$\lambda_1^{(\text{Nb}_3\text{Sn})}=120 \text{nm}$

SC substrate: clean Nb
$B_{\text{max}}^{(\text{Nb})}=240 \text{mT}$
$\lambda_2^{(\text{Nb})}=40 \text{nm}$

---

**assumption**

S layer: $\text{Nb}_3\text{Sn}$ (moderately dirty)
$B_c^{(\text{Nb}_3\text{Sn})}=540 \text{mT}$
$\lambda_1^{(\text{Nb}_3\text{Sn})}=120 \text{nm}$

SC substrate: clean Nb
$B_{\text{max}}^{(\text{Nb})}=200 \text{mT}$
$\lambda_2^{(\text{Nb})}=40 \text{nm}$

---

**assumption**

S layer: $\text{Nb}_3\text{Sn}$ (moderately dirty)
$B_c^{(\text{Nb}_3\text{Sn})}=540 \text{mT}$
$\lambda_1^{(\text{Nb}_3\text{Sn})}=120 \text{nm}$

SC substrate: clean Nb
$B_{\text{max}}^{(\text{Nb})}=B_{c1}^{(\text{Nb})}=170 \text{mT}$
$\lambda_2^{(\text{Nb})}=40 \text{nm}$
The optimum $d_s$ and the maximum field for NbN / I / Nb system, where Nb substrate is assumed to withstand up to 240, 200, and 170 mT.

**Assumption**

- **S layer:** NbN
  - $B_c^{(NbN)} = 230$ mT
  - $\lambda_1 = \lambda^{(NbN)} = 200$ nm
- **SC substrate:** clean Nb
  - $B_{\text{max}}^{(Nb)} = 240$ mT
  - $\lambda_2 = \lambda^{(Nb)} = 40$ nm

**Assumption**

- **S layer:** NbN
  - $B_c^{(NbN)} = 230$ mT
  - $\lambda_1 = \lambda^{(NbN)} = 200$ nm
- **SC substrate:** clean Nb
  - $B_{\text{max}}^{(Nb)} = 200$ mT
  - $\lambda_2 = \lambda^{(Nb)} = 40$ nm

**Assumption**

- **S layer:** NbN
  - $B_c^{(NbN)} = 230$ mT
  - $\lambda_1 = \lambda^{(NbN)} = 200$ nm
- **SC substrate:** clean Nb
  - $B_{\text{max}}^{(Nb)} = B_{c1}^{(Nb)} = 170$ mT
  - $\lambda_2 = \lambda^{(Nb)} = 40$ nm
$\eta < 1$
Evaluate the superheating field suppression factor, $\eta$

**Example: electropolished Nb**


We can find typical $\eta$ of EPed surface

The optimum $d_s$ and the maximum field for Dirty Nb / $I$ / Nb system, when the suppression factor due to nano-defects are given by $\eta=0.9$, 0.7, and 0.5. Nb substrate is assumed to withstand up to 170mT.

**assumption**

S layer: Dirty Nb

$B_{c}^{(Nb)}=200\text{mT}$

$\lambda_1=\lambda^{(dirty\text{Nb})}=180\text{nm}$

SC substrate: clean Nb

$B_{\text{max}}^{(Nb)}=B_{c_1}^{(Nb)}=170\text{mT}$

$\lambda_2=\lambda^{(Nb)}=40\text{nm}$
The optimum $d_s$ and the maximum field for $\text{Nb}_3\text{Sn} / I / \text{Nb}$ system, when the suppression factor due to nano-defects are given by $\eta=0.9, 0.7, \text{and } 0.5$. Nb substrate is assumed to withstand up to 170mT.

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**Optimum $d_s \sim 150\text{nm}$**
**Maximum field $\sim 430\text{mT}$**

**Optimum $d_s \sim 130\text{nm}$**
**Maximum field $\sim 350\text{mT}$**

**Optimum $d_s \sim 90\text{nm}$**
**Maximum field $\sim 270\text{mT}$**

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**assumption**

S layer: $\text{Nb}_3\text{Sn}$ (moderately dirty)

$B_{c}^{(\text{Nb}_3\text{Sn})}=540\text{mT}$

$\lambda_1=\lambda^{(\text{Nb}_3\text{Sn})}=120\text{nm}$

SC substrate: clean Nb

$B_{\text{max}}^{(\text{Nb})}=B_{c1}^{(\text{Nb})}=170\text{mT}$

$\lambda_2=\lambda^{(\text{Nb})}=40\text{nm}$
The optimum $d_s$ and the maximum field for NbN / I / Nb system, when the suppression factor due to nano-defects are given by $\eta = 0.9$, 0.7, and 0.5. Nb substrate is assumed to withstand up to 170mT.

**Assumption**

- **S layer**: NbN
  - $B_{c}^{(NbN)} = 230mT$
  - $\lambda_1 = \lambda^{(NbN)} = 200nm$
- **SC substrate**: clean Nb
  - $B_{\text{max}}^{(Nb)} = B_{c1}^{(Nb)} = 170mT$
  - $\lambda_2 = \lambda^{(Nb)} = 40nm$